

# Thermodynamic properties of San Carlos olivine at high temperature and high pressure

Chang Su<sup>1,2</sup> · Yonggang Liu<sup>1</sup> · Wei Song<sup>1</sup> · Dawei Fan<sup>1</sup> · Zhigang Wang<sup>3</sup> ·  
Hongfeng Tang<sup>1</sup>

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**Abstract** In this study, the thermal expansion and heat capacity of San Carlos olivine under high temperature and high pressure are reported. Combining accurate sound velocity data under different P–T conditions with density and heat capacity data at ambient pressure, the density, adiabatic bulk modulus, shear modulus, and most importantly, thermal expansion and heat capacity, of San Carlos are extracted to 14 GPa by a numerical procedure using classic thermodynamic relationships. These data are in agreement with published findings. To estimate the temperature gradient in the upper mantle, we also report the fitting equations of thermal expansion and heat capacity of San Carlos olivine as a function of both temperature and

pressure to the P–T condition of the 410 km discontinuity, which provide the thermodynamic properties with increasing depth in the Earth's interior.

**Keywords** San Carlos olivine · Thermodynamic property · Thermal expansion · Heat capacity · Temperature gradient

## 1 Introduction

To get a better understanding of the composition and dynamics of the mantle, it is important to determine the thermodynamic properties as a function of the depth of the common minerals in the Earth's interior (Akaoigi et al. 2007; Chen et al. 1996; Li et al. 2017). Thermodynamic parameters such as heat capacity and thermal expansion of mantle minerals are related to both geodynamics and mineral physics, and their values at high pressure are major factors that controlling the Earth's internal evolution (Yoneda et al. 2009).

Thermal expansion of mantle minerals largely depends on temperature and pressure, which can be deduced from experimental measurements on volume (Anderson 1967). Additionally, heat capacity at ambient pressure can be obtained using calorimetric experiments (Ashida et al. 1987; Barin et al. 1973; Watanabe 1982) or theoretical calculations (Akaoigi et al. 1984; Jacobs et al. 2017; Price et al. 1987). By using transient hot-wire method, the specific heat can be directly measured, but only in a low-pressure range (Andersson and Ross 1994; Nesbitt et al. 2017). Osako et al. (2004) presented a new method to obtain the high pressure heat capacity in terms of simultaneous thermal conductivity and thermal diffusivity measurement (Osako et al. 2004). Heat capacity of the solids is considered to be virtually independent of pressure

✉ Yonggang Liu  
liuyonggang@vip.gyig.ac.cn

Chang Su  
suchang@mail.gyig.ac.cn

Wei Song  
songweig@mail.gyig.ac.cn

Dawei Fan  
fandawei@vip.gyig.ac.cn

Zhigang Wang  
wangzg@caep.cn

Hongfeng Tang  
tanghongfeng@vip.gyig.ac.cn

<sup>1</sup> Key Laboratory of High-Temperature and High-Pressure Study of the Earth's Interior, Institute of Geochemistry, Chinese Academy of Sciences, Guiyang 550081, China

<sup>2</sup> University of Chinese Academy of Sciences, Beijing 100049, China

<sup>3</sup> National Key Laboratory of Shock Wave and Detonation Physics, Institute of Fluid Physics, Chinese Academy of Engineering Physics, Mianyang 621900, China

(Navrotsky 1995), and its pressure derivation is sometimes ignored. However, considering its importance in calorimetric study, it is still worth determining the heat capacity for common mantle minerals at high pressure and its pressure dependence.

Davis and Gordon introduced a numerical procedure which can be used to derive the equation of state (EoS) (Davis and Gordon 1967). This equation set is based on a series of classical thermodynamic relationships, allowing accurate determination of the volume of a liquid as a function of pressure and temperature from experimental adiabatic sound velocity. This method has been applied on other melts or solutions (Ayrinhac et al. 2015; Chorazewski et al. 2013; de Koker 2012; Su et al. 2017), thus proving its reliability. The accurate results show that this method is independent of any knowledge about compression in solid phases, and the sound velocity as a function of density follows a scaling law valid across the entire metallic state regime. But as far as we know, this numerical procedure is rarely used in single cell or polycrystalline minerals.

In order to test the feasibility of this method on minerals, we chose San Carlos olivine as our research material. San Carlos olivine is the ultramafic inclusion that is located in San Carlos, Arizona with a Mg:Fe ratio of 9:1 [ $(\text{Mg}_{0.9}\text{Fe}_{0.1})_2\text{SiO}_4$ ] (Frey and Prinz 1978), which is the approximate composition of the peridotites in the mantle (Kojitani et al. 2016). Olivine is a major component of the upper mantle (Liu et al. 2005), and the determination of its thermodynamic properties is one of the most important themes in mineral science (Jianping et al. 1995). The phase transition of olivine into its high pressure polymorph wadsleyite at  $\sim 13\text{--}14$  GPa is considered to be the most likely cause of the discontinuity at 410 km depth, from which the temperature of the 410 km discontinuity can be deduced from the thermodynamic changes. Because of this, numerous amount of studies have been carried out that provide the elastic properties of San Carlos olivine in a wide pressure and temperature range (Abramson et al. 1997; Darling et al. 2004; Isaak 1992; Liu et al. 2005; Mao et al. 2015; Zhang and Bass 2016).

In this study, using our original research and data from previous studies, we report the molar volume, thermal expansion, adiabatic bulk modulus and shear modulus of San Carlos olivine at high temperature and high pressure extracted with the numerical calculation. Then, we compare our findings with previous scholarship. Specifically, we present the thermal expansion and heat capacity of San Carlos olivine to 14 GPa and provide an equation that relates to both temperature and pressure. Finally, using the thermal expansion and heat capacity data of San Carlos olivine, we propose the temperature gradient of the upper mantle.

## 2 Calculation procedure

### 2.1 Thermodynamic calculation

We used data provided in Davis and Gordon (1967) to determine the calculation procedure and obtain the EoS of San Carlos olivine and its thermodynamic parameters as a function of temperature and pressure. This method uses classical thermodynamic relations to extract the density variation as a function of pressure and temperature from the adiabatic sound velocities. The fundamental equations are described below.

First, the thermal expansion ( $\alpha$ ) is defined as:

$$\alpha = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_P \quad (1)$$

Then, according to the definition of heat capacity ( $C_P$ ), its variation with pressure can be evaluated as Eq. (2), which is based on the Maxwell relation:

$$\left( \frac{\partial C_P}{\partial P} \right)_T = -\frac{T}{\rho} \left[ \alpha^2 + \left( \frac{\partial \alpha}{\partial T} \right)_P \right] \quad (2)$$

Furthermore, the partial differential equation about the density at high pressure can be derived from the relationship between the isothermal and adiabatic bulk modulus:

$$\left( \frac{\partial \rho}{\partial P} \right)_T = \frac{1}{v^2} + \frac{T \alpha^2}{C_P} \quad (3)$$

where  $v$  stands for the sound velocity, and in the past only P-wave velocity was used in the previous works on liquids (Ayrinhac et al. 2014, 2015; Su et al. 2017). To test whether this method could be applied to solids or not, we made a change on Eq. (3) to use the bulk velocity ( $v_B$ ) instead:

$$v_B = \left( v_p^2 - \frac{4}{3} v_s^2 \right)^{\frac{1}{2}} \quad (4)$$

To start the calculation, first we calculated  $\rho$  as a function of temperature at zero pressure  $\rho(P_0, T)$  and  $\alpha$  at ambient pressure with Eq. (1). It is also necessary to determine the values of the heat capacity with increasing temperature  $C_P(P_0, T)$  in order to derive the approximate  $\rho$  at an arbitrary reference pressure using Eq. (2). The resulting  $\rho$  is used to update the value of  $\alpha_P$  and  $C_P$  at the same pressure with Eqs. (1) and (2). Iteration of this loop leads to converged values of  $\rho$ ,  $\alpha_P$  and  $C_P$  at high pressure conditions. Furthermore, with the deduced results, the adiabatic bulk modulus ( $K_S$ ) and shear modulus ( $G$ ) can also be obtained:

$$K_S = \rho \left( v_p^2 - \frac{4}{3} v_s^2 \right) \quad (5)$$

$$G = \rho v_s^2 \quad (6)$$

## 2.2 Thermodynamic data

### 2.2.1 Sound velocity at high temperature and high pressure

The sound velocities of San Carlos olivine have been investigated using different experimental methods (Abramson et al. 1997; Darling et al. 2004; Isaak 1992; Liu et al. 2005; Mao et al. 2015; Zhang and Bass 2016). Isaak first reported the high temperature elastic moduli for Fe-bearing olivine, which provided various parameters as a function of temperature at ambient pressure (Isaak 1992). Then Abramson et al. (1997) measured the sound velocities up to 17 GPa at room temperature in a diamond anvil cell, and the results were consistent with those given by Darling et al., which were obtained with a Kawaii type multi-anvil apparatus (Darling et al. 2004). The highest temperature and pressure range data available to date was determined by Zhang and Bass, who used the Brillouin spectroscopy with CO<sub>2</sub> laser-heating and calculated the sound velocities of San Carlos olivine to 16.5 GPa at room temperature and 12.8 GPa at 1300 K (Zhang and Bass 2016).

Though this research provides useful data on the elastic properties of the olivine in a wide range of P–T conditions, what is needed is more detailed sound velocity data on a smaller interval of temperature and pressure. The data published by Liu et al. (2005) and Mao et al. (2015) provide sound velocities at both high temperature and high pressure conditions. With a DIA type large volume apparatus, Liu et al. (2005) measured the P-wave and S-wave sound velocities of the polycrystalline San Carlos olivine to 8 GPa and 1073 K, while Mao et al. (2015) analyzed natural single crystals using the high P–T Brillouin measurements, which provided a smaller difference between the experimental data and the isothermal fitting results. Therefore, in this paper, we used Mao's sound velocities of San Carlos olivine.

Using the  $v_p$  and  $v_s$  data from Isaak (1992), we determined  $v_p$  and  $v_s$  to analyze the high P–T data to ambient pressure, thus permitting us to obtain the P–T–v relationship as Eq. (7), whose form is based on the work by Ayrinhac et al. (2015):

$$v(P, T) = \sum_{i,j} a_{ij} T^i P^j \quad (7)$$

The fitting coefficients are shown in Table 1, with a reduced  $\chi^2$  as 2.35 for  $v_p$  and 3.85 for  $v_s$ , respectively.

### 2.2.2 Density at ambient pressure

The densities of San Carlos olivine at room temperature and ambient pressure are given as 3353 kg m<sup>-3</sup> in Isaak

(1992), which were measured using the Archimedes immersion technique. Then Abramson et al. (1997) reported the density as 3355 kg m<sup>-3</sup> using the buoyancy method, and this result is close to the value of 3360 kg m<sup>-3</sup> based on the lattice constants. Zha et al. (1998) measured the density with X-ray diffraction and found a value of 3343 kg m<sup>-3</sup>, which is in agreement with the latest data 3341 kg m<sup>-3</sup> using the same method (Zhang and Bass 2016). For the  $\alpha$ , here we chose  $\alpha = 3.304 \times 10^{-5} + 0.742 \times 10^{-8}T - 0.538T^{-2} K^{-1}$  by Fei (1995) based on the experimental data of Suzuki (1975). Therefore, a reliable extrapolation of  $\rho$  to high temperature can be constrained by previous studies:

$$\rho_0(T) = 3371.20 - 8.63 \times T + 2.88 \times T^2 \quad (8)$$

with  $\rho_0$  in kg m<sup>-3</sup> and T in K.

### 2.2.3 Heat capacity at ambient pressure

The heat capacity of San Carlos olivine at ambient pressure was also provided by Isaak (1992), which was determined from the end-member heat capacity data from Barin et al. (1973). Richet presented a way to obtain the heat capacity of the silicate glass with increasing temperature (Richet 1987), and he found that the value depended on the composition of the silicate glass. In Richet's (1987) fitting equation, the temperature limit was lower than 800 K. In our calculation, we determined the values of heat capacity to be greater than 800 K, so in this work, we used Isaak's ambient pressure heat capacity data. Berman and Brown gave a revised formula that represented and extrapolated the heat capacity with increasing temperature (1985), which was based on the lattice vibration theory, hence the fitting equation is:

$$C_P = 995.1 + 1343T^{-0.5} - 2.887 \times 10^7 T^{-2} - 6.166 \times 10^{-2} T^{-3} \quad (9)$$

with  $C_P$  in J kg<sup>-1</sup> K<sup>-1</sup> and T in K.

With the data listed above, we derived the density, thermal expansion, heat capacity, adiabatic bulk modulus and shear modulus of San Carlos olivine under different temperature and pressure conditions.

## 3 Results and discussion

### 3.1 EoS and elastic properties comparisons

For the EoS of San Carlos olivine, the room temperature volume was measured using impulsively stimulated laser scattering to 17 GPa by Abramson et al. (1997) and later, using Brillouin spectroscopy to ~ 30 GPa by Zha et al.

**Table 1**  $a_{ij}$  coefficients for Eq. (7) with  $v$  in  $\text{m s}^{-1}$ ,  $T$  in K and  $P$  in GPa

i/j	0	1	2
$v_p$			
0	$8364.17 \pm 1.28$	$109.24 \pm 0.12$	$-2.47 \pm 0.01$
1	$-0.44 \pm 0.01$	$(-3.35 \pm 0.12) \times 10^{-3}$	—
2	—	—	—
$v_s$			
0	$4862.27 \pm 0.77$	$42.83 \pm 0.07$	$-1.68 \pm 0.01$
1	$(-2.81 \pm 0.02) \times 10^{-1}$	$(8.37 \pm 0.01) \times 10^{-3}$	—
2	$(-2.26 \pm 0.19) \times 10^{-5}$	—	—

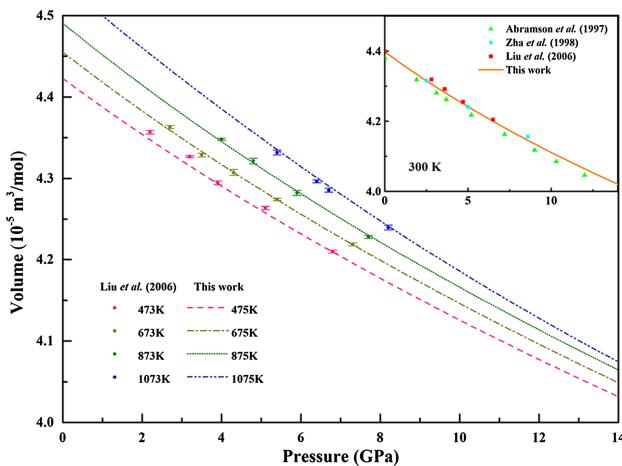
(1998). The high temperature volume data was determined to 1073 K and 8 GPa using in situ synchrotron X-ray diffraction measurements (Liu and Li 2006). Figure 1 illustrates the molar volume of San Carlos olivine with increasing temperature and pressure, as well as the former results as comparisons.

From Fig. 1, we can see that the calculated results generally agree with the previous experimental data (Abramson et al. 1997; Liu and Li 2006; Zha et al. 1998). At room temperature, our results show a similar trend with Abramson's results, with a difference of  $\sim 0.4\%$ . The largest separation between our result and the other two works is 0.3% at 2.8 GPa. At high temperatures, our theoretical data is consistent with the experimental data, with a reduced  $\chi^2 = 7.63$ . As we mentioned above, the ambient pressure density data we used were obtained from theoretical calculations, so its difference from the experimental data was predicted to be large. However, with the increasing pressure, we can see that the separations became smaller, especially over  $\sim 4$  GPa. Hence, though different measurements would likely give data with different uncertainties, because of the self-consistency of the

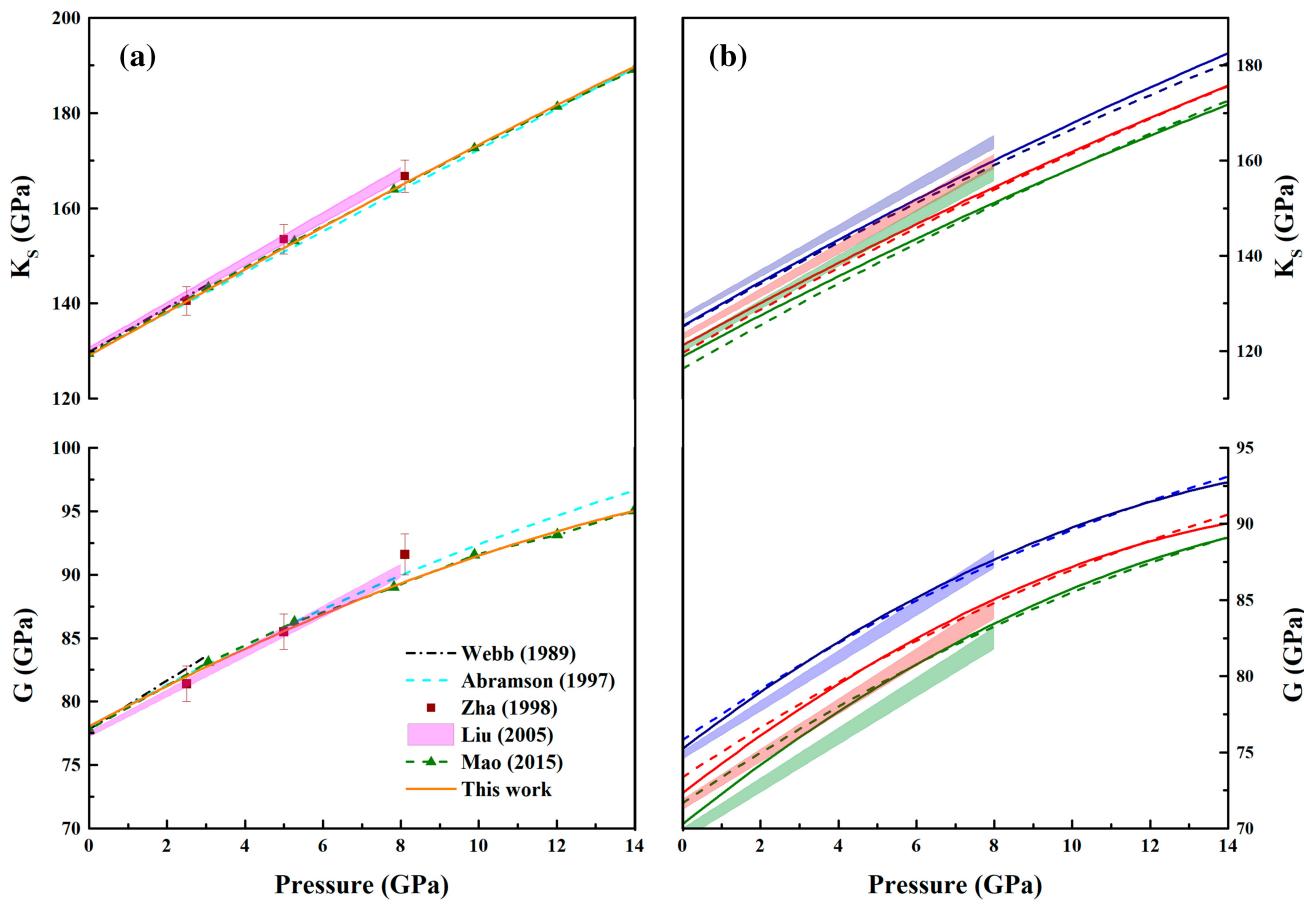
thermodynamic parameters, the calculated results could be fixed when using this theoretical method, which would finally provide reliable results.

The other quantities derived from our calculation are adiabatic bulk modulus and shear modulus. Based on Hashin–Shtrikman bounds, the aggregate  $K_S$  and  $G$  of San Carlos olivine have been determined to 3 GPa (Webb 1989), 17 GPa (Abramson et al. 1997) and 32.5 GPa (Zha et al. 1998) at room temperature using various techniques. For high temperature data, Liu et al. (2005) provide the values not only of  $K_{S0}$ ,  $G_0$  and their pressure derivation, but also their temperature derivation to 8 GPa and 1073 K (Liu et al. 2005). Mao et al. (2015), whose sound velocities we utilized in our calculation, calculated the elastic moduli of San Carlos olivine to  $\sim 18$  GPa and 900 K. With the density data of San Carlos olivine as a function of both temperature and pressure, the values of  $K_S$  and  $G$  can be obtained with Eqs. (5)–(6), thus our calculated results are shown in Fig. 2 along with the previous works.

Figure 2a, b illustrate the  $K_S$  and  $G$  with increasing pressure at room temperature and high temperature, respectively. Generally, for  $K_S$ , its values follow a nearly linear increase with pressure under same temperature, whereas  $G$  exhibits a downward trend towards higher pressure. In Fig. 2a, our results agree with the former ones and are close to Mao's results, which gives a largest separation as 0.4% at  $\sim 14$  GPa for both  $K_S$  and  $G$ . This likely occurs due to our use of Mao's sound velocity data during our calculation. For  $K_S$ , all of the works consistently agree with each other except for Liu et al. (2005)'s data, whose slope is steeper than the others. For  $G$ , the separations get larger above  $\sim 5$  GPa, and our slope gets smoother comparing to the others. In Fig. 2b, the differences seem to be quite large from different methods, especially under  $\sim 6$  GPa, but then the consistency between our results and Mao et al.'s (2015) results improves with increasing pressure.



**Fig. 1** Molar volume of San Carlos olivine as a function of pressure at different temperatures [curves: theoretical results from this work; solid circles: experimental results from Liu et al. (2005); insert: room temperature data with previous work]



**Fig. 2** Comparison of adiabatic bulk modulus and shear modulus. **a** Room temperature condition. **b** High temperature condition. Blue: 500 K; red: 750 K; green: 900 K; filled areas: Liu et al. (2005); dash lines: Mao et al. (2015); solid lines: this work

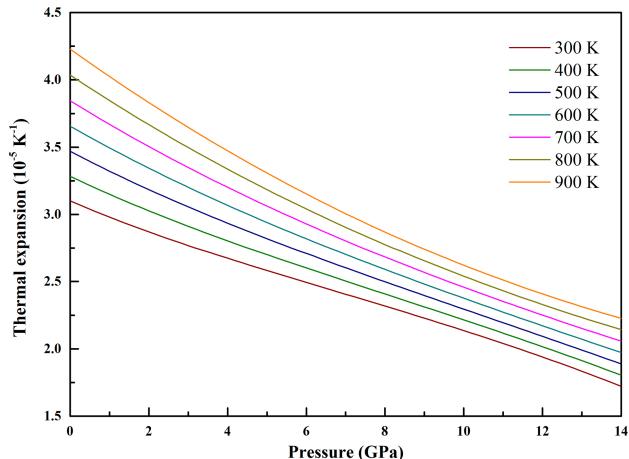
### 3.2 Thermal expansion and heat capacity at high temperature and high pressure

For the thermodynamic parameters deduced from Eqs. (1)–(4), here we present the thermal expansion and heat capacity of San Carlos olivine under different temperatures with increasing pressure in Figs. 3 and 4, respectively.

The thermal expansions of minerals at high pressure are usually related to their volumes by the Anderson-Grüneisen parameter ( $\delta_T$ ), which describes the degree of decrease in  $\alpha$  by compression (Anderson 1966). The expression is:

$$\frac{\alpha}{\alpha_0} = \left( \frac{V_P}{V_0} \right)^{\delta_T} \quad (10)$$

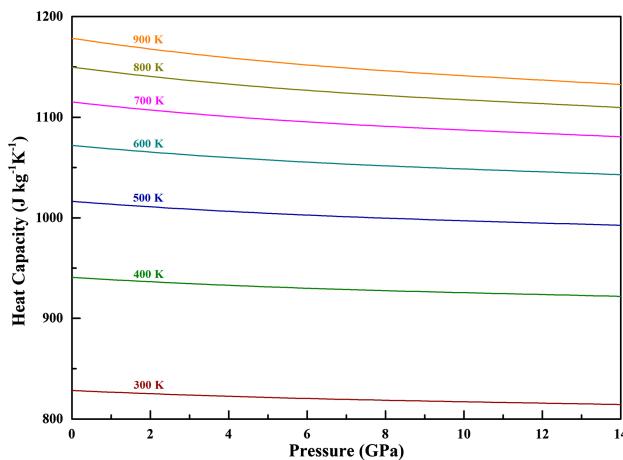
where  $\alpha_0$  is the thermal expansion at ambient pressure and  $V_0$ ,  $V_P$  are volumes at ambient pressure and high pressure, respectively. In this work, the value of  $\delta_T$  is calculated as  $7.15 \pm 0.6$ , which is similar as forsterite ( $7.2 \pm 0.3$ , Katsuma et al., 2010). Furthermore, the product of thermal expansion and isothermal bulk modulus ( $\alpha K_T$ ), which can be considered volume-independent within the investigated pressure and temperature range, is  $4.02 \pm 0.35 \times 10^{-3}$ ,



**Fig. 3** Thermal expansion of San Carlos olivine as a function of pressure at different temperatures

and this result is close to the value given by Liu and Li ( $4.08 \pm 0.10 \times 10^{-3}$ , 2006).

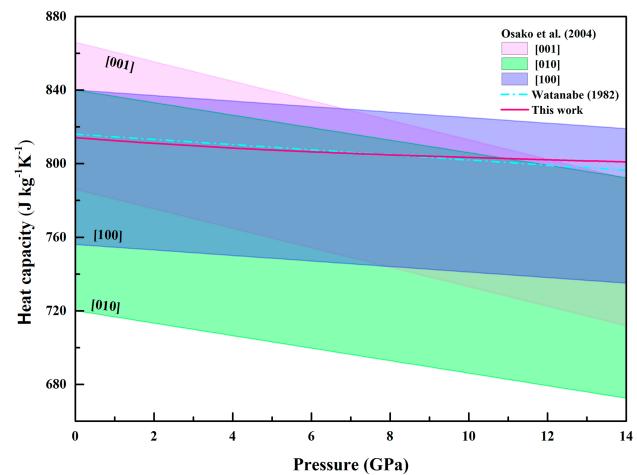
Most of the data about the heat capacity of olivine or other minerals are obtained at ambient pressure using calorimeters (Akaogi et al. 2007; Robie et al. 1982;



**Fig. 4** Heat capacity of San Carlos olivine as a function of pressure at different temperatures

Watanabe 1982), and the sources at high pressure are still rare. Though the heat capacities have been measured in the piston cylinder (Andersson and Ross 1994; Hashimoto et al. 2006), the pressure range was limited to 2 GPa. Actually, the experimental method to measure the heat capacity directly continues to be a problem because of the difficulty of obtaining calorimetric measurements in high temperature and high pressure (Yoneda et al. 2009). In mineralogy, heat capacity is usually related to thermal conductivity ( $\lambda$ ) and thermal diffusivity ( $\kappa$ ), hence in 2004, a method to obtain the heat capacity was developed by Osako et al. (2004), which simultaneously measured  $\lambda$  and  $\kappa$  at high temperature and high pressure, thus permitting for the heat capacity to be calculated. Osako et al. (2004) measured the  $\lambda$  and  $\kappa$  of San Carlos olivine, which provided the pressure derivation of its heat capacity ( $dC/dP$ ) and the values of the  $C_P/dP$  changed for different crystal axis. Also, the value of  $dC_P/dP$  deduced from the ambient pressure data was also presented (Suzuki 1975; Watanabe 1982).

In Fig. 5, we illustrate the heat capacities with increasing pressure at room temperature, which are calculated using Eq. (12) and the values from Watanabe (1982) and Osako et al. (2004). At ambient pressure, our  $C_P|_{P=0}$  is  $814 \text{ J kg}^{-1}\text{K}^{-1}$ , which is close to the values from Watanabe's result. Also in combining data of Watanabe (1982) and Suzuki (1975), in Fig. 5, we can see that the variations among the slopes from different axis are large, except for the [1 0 0] one, whose slope is  $-1.5$ , thus making it quite similar to Watanabe's (1982) results. Meanwhile, our result is consistent with that of Watanabe, and the slope is  $dC_P/dP = 3.67 \times 10^{-2}P - 1.43$ , where  $P$  is in GPa. Since the error of the heat capacities from Osako et al. (2004) is 6%, our results still coincide with theirs (Osako et al. 2004, Watanabe, 1982).



**Fig. 5** Heat capacity vs pressure at room temperature with previous work

### 3.3 Temperature gradient to 410 km

One of the most important applications of thermal expansion and heat capacity at high pressure is used to estimate the temperature gradient in the Earth's interior. Since conductive and radiative heat transfer can be ignored because of the small thermal conductivity of mantle minerals, the temperature gradient is considered to be nearly adiabatic. Therefore, the adiabatic temperature gradient can be expressed as:

$$\left(\frac{\partial T}{\partial z}\right)_S = \frac{\alpha g T}{C_P} \quad (11)$$

where  $z$  is the depth,  $g$  is the gravitational acceleration (Katsura et al. 2010).

To calculate the temperature gradient in the upper mantle, we extrapolate the thermal expansion and heat capacity to the pressure and temperature range at the 410 km discontinuity in the mantle. Considering the temperature range in this study, it should be an adequate approximation to treat thermal expansion as linear in temperature at ambient pressure (Katsura et al. 2010). For heat capacity, its variation with increasing temperature at ambient pressure can be fitted using the model from Bereman and Brown (1985). As seen in Figs. 3 and 4, both thermal expansion and heat capacity at high pressure can clearly be presented by a polynomial fitting equation, thus with the data derived from Eqs. (1)–(4), the thermal expansion and heat capacity of San Carlos olivine as a function of both temperature and pressure can be expressed as Eqs. (12) and (13).

$$\alpha(P, T) = a_0 + a_1 T + a_2 P + a_3 PT + a_4 P^2 \quad (12)$$

$$C_P = c_0 + c_1 T^{-0.5} + c_2 T^{-2} + c_3 P + c_4 P^2 \quad (13)$$

The fitting coefficients for Eqs. (12) and (13) are listed in Tables 2 and 3, respectively. The misfit for Eq. (12) is less than 2 and 1.6% for Eq. (13).

We adopted the pressure gradient as 0.034 GPa km<sup>-1</sup> from previous work (Matsui et al. 2000) and the gravitational acceleration from PREM model; therefore, the temperature gradient can be calculated from Eq. (11).

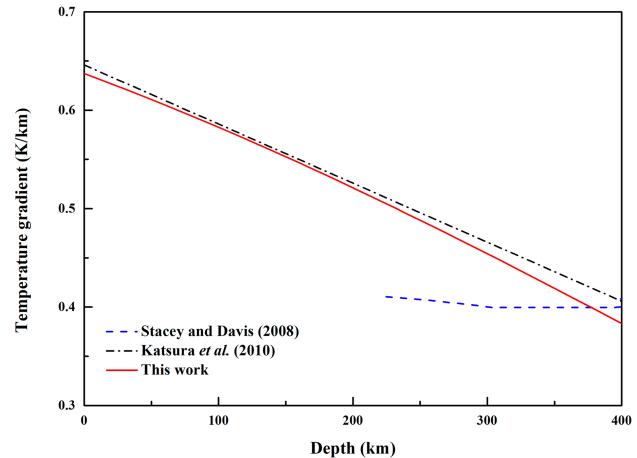
The estimated temperature gradient is shown in Fig. 6 along with the former results (Katsura et al. 2010; Stacey and Davis 2008). Figure 7 illustrates the thermal expansion variation in the mantle. The gradient is  $\sim 0.64$  K km<sup>-1</sup> at the top of the asthenosphere, then decreases to  $\sim 0.37$  K km<sup>-1</sup> at the 410 km discontinuity. In Fig. 7, the trend in our results generally agrees with that of Katsura et al. (2010) in concluding that the values decrease with increasing depth. The thermal expansion given by Stacey and Davis (2008) and Katsura et al. (2010), respectively, was for forsterite, which is Fe-free olivine, and the thermal expansion for San Carlos olivine is approximately 10% higher than that of forsterite, which might be caused by the content of Fe.

#### 4 Conclusion

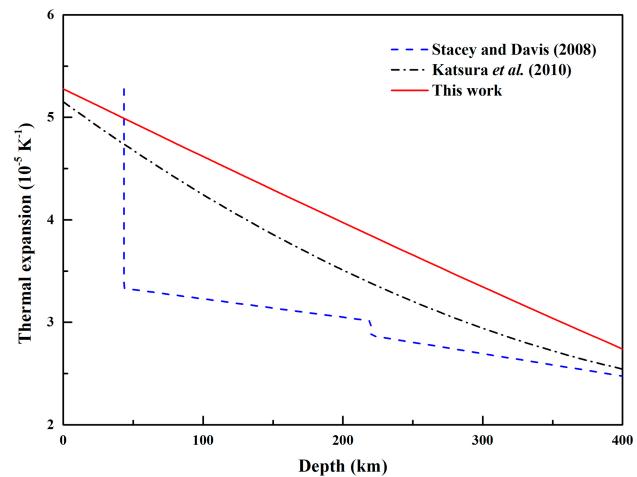
In this work, we used a numerical method, which was applied to liquids only, to calculate various parameters of San Carlos olivine. Matching the previous data of heat capacity and density at ambient pressure with sound velocities at high pressure, we derived the molar volume, adiabatic bulk modulus, and shear modulus as a function of temperature and pressure, and then we extrapolated the pressure range to about 14 GPa, which is the approximate pressure of the 410 km discontinuity. The calculated results agreed with the former experimental data and proved the feasibility of our theoretical method. Most importantly, our theoretical method can be used to determine the thermal expansion and heat capacity of minerals at high pressure, which are hard to measure through experimentation. We not only deduced a fitting equation of thermal expansion and heat capacity of San Carlos olivine as a function for both temperature and pressure, but also

**Table 3** Fitting coefficients for Eq. (13), P in GPa, T in K and C<sub>P</sub> is in J kg<sup>-1</sup> K<sup>-1</sup>

Coefficients	$c_0$	$c_1 \times 10^{-4}$	$c_2 \times 10^{-6}$	$c_3$	$c_4 \times 10^2$
Values	1585	- 1.238	- 3.139	- 3.184	8.414
±	2	0.004	0.055	0.015	0.377



**Fig. 6** Temperature gradient with increasing depth



**Fig. 7** Thermal expansion variation of San Carlos olivine with increasing depth

proposed the variations of heat capacity with increasing pressure  $\partial C_P / \partial P = 3.67 \times 10^{-2}P - 1.43$ , which are useful for making decisions in a thermodynamic analysis. Finally, we presented the temperature gradient to the 410 km discontinuity using the thermodynamic parameters that we derived, which provided the variations of the thermodynamic properties in the mantle.

**Table 2** Fitting coefficients for Eq. (12), P in GPa, T in K and  $\alpha$  in K<sup>-1</sup>

Coefficients	$a_0 \times 10^{-5}$	$a_1 \times 10^{-8}$	$a_2 \times 10^{-5}$	$a_3 \times 10^{-10}$	$a_4 \times 10^{-8}$
Values	2.627	1.695	- 0.103	- 7.907	2.286
±	0.023	0.036	- 0.004	0.435	0.212

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